PHY489 Week 7:
Feynman Rules (Toy Model)

Calculating amplitudes $i\mathcal{A}_{if}$? $\longrightarrow$ Feynman Rules!
Diagrammatic Perturbation Theory!

- $i\mathcal{A}_{if}$ determined by interaction Lagrangian $\mathcal{L}$ and obviously this is outside the scope of this course...

i.e. $$\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\not{D} = \gamma^{\mu}D_{\mu} = \gamma^{\mu}(\partial_{\mu} + iA_{\mu})$$

Rules:

1. Notation:

Label incoming / outgoing momenta $p_1, p_2, \dots, p_N$.

Label internal momenta $q_1, q_2, \dots, q_3$.

2. Coupling Vertices:

For each vertex, include a factor of

$(-ig)(2\pi)^4 \delta^{(4)}\left(\sum_i k_i\right)$, where g is the constant specifying the strength of the interaction and the k_i's are the 4-momenta (p_k or q_k) flowing in or out of that vertex.

3. Propagators.

For each internal line of momentum q, include the field contraction propagator:

i, j isospin indices

$$i\Delta_{ij}(q) = \frac{i\delta_{ij}}{q^2 - m^2 + i\epsilon} \qquad \text{[spin 0]}$$

wick rotation

α, β spinor indices

$$iS_{\alpha\beta}(q) = \frac{i(\not{q} + m)_{\alpha\beta}}{q^2 - m^2 + i\epsilon} \qquad \text{[spin } \tfrac{1}{2}\text{]}$$

μ, ν polarization indices

$$i\Delta_{\mu\nu}(q) = \frac{i}{q^2}\left\{-g_{\mu\nu} + (1-\alpha)\frac{q_\mu q_\nu}{q^2}\right\} \qquad \text{[Spin 1 Photon/Gluon]}$$

$\alpha = 1$ Feynman/Lorentz Gauge

$\alpha = 0$ Landau Gauge

$$i\Delta_{\mu\nu}(q) = \frac{i}{q^2 - m^2 + i\epsilon}\left\{-g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2}\right\} \qquad \text{[Spin 1 Massive Vector Boson]}$$

μ, ν polarization indices.

4. Unspecified Internal Momenta

Integrate over each internal 4-momentum (q not fixed by energy-momentum conservation) with a weight

$$\int \frac{d^4 q}{(2\pi)^4}.$$

- closed loops
- internal (propagator) momenta (the latter, with δ-fcts, shows the momentum value passing through the internal line.

5. Closed Fermion Loops:

For each closed fermion loop, include a (-1) factor.

6. Incoming, Outgoing Particles:

[Fermions]

Assemble incoming fermion spinors, vertex operators, propagators, and outgoing fermion spinors in order along each fermion line to make a well-formed matrix element. In particular:

- Multiply from the left by $\bar{u}(\vec{p}_-, s_-)$ for each outgoing fermion with momentum $\vec{p}_-$ and spin s_-.
- Multiply from the right by $u(\vec{k}_-, s_-)$ for each incoming fermion with momentum $\vec{k}_-$ and spin s_-.
- Multiply from the right by $v(\vec{p}_+, s_+)$ for each outgoing antifermion with momentum $\vec{p}_+$ and spin s_+.
- Multiply from the left by $\bar{v}(\vec{k}_+, s_+)$ for each incoming antifermion with momentum $\vec{k}_+$ and spin s_+.

[Photons, Vector Bosons]

For photons and vector bosons, construct well-formed vector products by saturating (contracting) any free vector polarization indices μ on current operators γ^μ by:

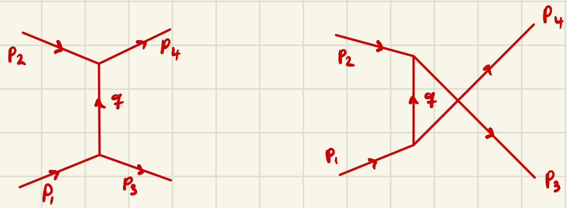
- Multiply by ε_μ^* for each <u>outgoing particle</u> with polarization index μ.
- Multiply by ε_μ for each <u>incoming particle</u> with polarization index μ.

These rules should be sufficient for this class, however there are many additional rules which we must account for when we do renormalization / electroweak theory / closed boson bubbles / etc.

Lorentz invariance of an amplitude $i\mathcal{A}_{fi}$ is required and hence enforced by the Feynman rules.

Consider the scattering example $AA \to BB$:

Two possible arrangements are possible for momentum transfer within the propagator, and we must account for both via a sum:



$q = p_1 - p_3$ $\qquad$ $q = p_1 - p_4$

(these aren't fermions - arrows are just indicating the flow of momentum)

$$i\mathcal{A}_{fi} = i\mathcal{A}_{fi}^{(1)} + i\mathcal{A}_{fi}^{(2)}$$

$$= \left\{ \frac{g^2}{(p_1 - p_3)^2 - m_q^2 + i\epsilon} + \frac{g^2}{(p_1 - p_4)^2 - m_q^2 + i\epsilon} \right\}$$

and we must have $S = \frac{1}{2!}$ for identical particles BB.